# Introduction to Categorical Logic
## 80-514/814
### Suggested Topics for Student Projects

Updated: April 1, 2024

This document will grow as we go along. Come talk to me for more information about any of these topics. And feel free to suggest others!

Chapter 1: Algebraic Theories

- Lawvere duality: Finish the proof of duality for Cauchy complete FP categories.

    - Adamek, Lawvere, Rosicky: On the duality between varieties and algebraic theories, Algebra Universalis, 2003.
    - Adamek, Rosicky, Vitale: Algebraic theories, Cambridge University Press, 2010.

- Gabriel-Ulmer duality: Extend Lawvere duality to finite limit categories.

    - Makkai, Pitts, Some results on locally finitely presentable categories, Transactions of the AMS 1987.
    - Adamek, Rosicky, Vitale: Algebraic theories, Cambridge University Press, 2010.

- Stone-type dualities for commutative rings, distributive lattices, Heyting algebras, etc.

    - P.T. Johnstone, Stone Spaces, Cambridge University Press, 1982.

– M. Makkai and G. Reyes, Completeness results for intuitionistic and modal logic in a categorical setting, Annals of Pure and Applied Logic, Volume 72, Issue 1, 10 March 1995, Pages 25–101.

Chapter 2: Propositional Logic

- Priestly duality for distributive lattices
  - Clark and Davies, Natural Dualities for the Working Algebraist, Cambridge University Press, 1998.
  - P.T. Johnstone, Stone Spaces, Cambridge University Press, 1982.
- Frames, Locales, complete Heyting algebras (course notes)
- Modal Logic: various references, just ask (including a CMU MS thesis by H.-C. Kotzsch).
  - Kripke semantics for classical modal logic
  - Topological semantics for modal logic: S. Awodey and K. Kishida, Topology and Modality: The Topological Interpretation of First-Order Modal Logic, Review of Symbolic Logic, 2008.
  - What is intuitionistic S4 modal logic?
  - Modal propositional logic: McKinsey-Tarski topological completeness.
  - Gödel translation of IPC into classical modal PC.
- Bi-Heyting logic
  - F.W. Lawvere, Intrinsic Co-Heyting Boundaries and the Leibniz Rule in Certain Toposes, in A. Carboni, M. Pedicchio, G. Rosolini (eds.), Category Theory - Como 1990, LNM 1488 Springer Heidelberg 1991.
  - Gonzalo E. Reyes, Houman Zolfaghari, Bi-Heyting Algebras, Toposes and Modalities, J. Phi. Logic 25 (1996) pp. 25–43.
  - Kripke models of bi-Heyting logic (CMU MS thesis by J. Winkler).

Chapter 3: First-Order Logic

The following are discussed in the course notes and proved in Johnstone's Elephant [**?**].

- Freyd's embedding theorem for regular and coherent categories
- Kripke-Joyal semantics for IFOL in presheaves
- Completeness of K-J semantics using Joyal's embedding theorem
- Kripke completeness using the Diaconescu cover

Chapter 4: Type Theory

- Lambda-calculus and CCCs
  - D.S. Scott. Relating theories of the $\lambda$-calculus. In R. Hindley and J. Seldin, editors, To H.B. Curry: Essays in Combinatory Logic, Lambda Calculus and Formalisms, pp. 403–450. Academic Press, 1980.
  - D.S. Scott, Lambda Calculus: Some Models, Some Philosophy, Studies in Logic and the Foundations of Mathematics, Volume 101, 1980, pp. 223–265
- Completeness for CCCs (my course notes).
- Kripke completeness of the $\lambda$-calculus (ask for references).