Applicant: **Awodey, Steve**
Organisation: **University of Cambridge**
Funding Sought: **£86,786.00**

RSWVF\25\R2\1025

Homotopy Type Theory and the Formalization of Mathematics

This proposal advances recent discoveries in the foundations of mathematics with far-reaching applications for the certainty and precision of mathematics, as well as for the everyday work of mathematicians and other scientists. Homotopy Type Theory is an emerging field combining logic, mathematics, and computer science and employing a fundamentally new approach based on primitive higher-dimensional structures and including new principles of reasoning not directly available in conventional foundations. Its applications include providing powerful and flexible computational tools that facilitate the large-scale formalization of mathematics.
Standard mathematical proofs are still just arguments in words; famous examples of erroneous published proofs abound. Recent advances in high-speed computing have permitted the development of powerful interactive theorem provers to aid in the verification of mathematical proofs. The current project employs a new interpretation of type theory recently discovered by the PI and Fields medalist Vladimir Voevodsky, which brings such systems much closer to everyday mathematical practice.
The PI also proposes to teach a graduate level course based on a current textbook in progress. He will also organize and lead a working group devoted to an ongoing formalization project developing a new interactive proof assistant for Homotopy Type Theory based on the popular Lean Proof assistant that is being developed by an international team based at Carnegie Mellon University.
These activities are intended to strengthen the interaction and cooperation between the Cambridge and CMU research groups involved in related work. Future exchanges and collaborations among faculty, postdoctoral, and doctoral researchers are expected to result.

RSWVF\25\R2\1025

Homotopy Type Theory and the Formalization of Mathematics

Section 1 - Understanding our promotion

How did you hear about this Scheme?

⦿ Colleague/Friend

Section 2 - Contact Details

PRIMARY APPLICANT DETAILS

Title	Professor
Name	Steve
Surname	Awodey
Tel (Mobile)	(+1) 412-352-8051
Email (Work)	awodey@cmu.edu
Address	Carnegie Mellon University 5000 Forbes Ave Pittsburgh 15213 United States of America (the)

COLLABORATOR DETAILS

Role	Head of Department
Name	Alastair
Surname	Beresford
Website (Work)	www.cst.cam.ac.uk
Tel (Work)	01223 763500
Email (Work)	hod@cst.cam.ac.uk
Address	Department of Computer Science and Technology, William Gates Building JJ Thomson Avenue Cambridge Cambridgeshire CB3 0FD United Kingdom of Great Britain and Northern Ireland (the)

Role	Research Support
Name	CST
Surname	Grants
Tel (Work)	07907865670
Email (Work)	research-grants@cst.cam.ac.uk
Address	WGB, JJ Thomson Avenue Cambridge CB3 0US United Kingdom of Great Britain and Northern Ireland (the)

Role	Nominated Referee (2)
Title	Professor
Name	Glynn
Surname	Winskel
Organisation	Queen Mary University of London
Tel (Mobile)	07397868133
Email (Work)	g.winskel@qmul.ac.uk
Address	Mile End Road LONDON E1 4NS United Kingdom of Great Britain and Northern Ireland (the)

Role	Nominated Referee (1)
Title	Professor
Name	Emily
Surname	Riehl
Tel (Mobile)	+16173889118
Email (Work)	eriehl@jhu.edu
Address	3400 N Charles Street Department of Mathematics Baltimore MD 21218 United States of America (the)

GMS ORGANISATION

Type	University or Higher Education Institute
Name	University of Cambridge
Email	royalsoc.applications@rsd.cam.ac.uk
Address	The Old Schools Trinity Lane CAMBRIDGE CB2 1TN

Type	University or Higher Education Institute
Name	Queen Mary University of London
Email (Work)	researchgrants@qmul.ac.uk
Address	Mile End Road LONDON E1 4NS United Kingdom of Great Britain and Northern Ireland (the)

Section 3 - Eligibility Criteria

Please confirm whether you meet the eligibility criteria for the Royal Society Wolfson Fellowships programme as follows:

Do you either currently hold a permanent contract or have received a firm offer to take effect from the start date of the award?

◉ Yes

Have you previously held a Royal Society Wolfson Research Merit award?

⦿ No

Lay Summary

This proposal advances recent discoveries in the foundations of mathematics with far-reaching applications for the certainty and precision of mathematics, as well as for the everyday work of mathematicians and other scientists. Homotopy Type Theory is an emerging field combining logic, mathematics, and computer science and employing a fundamentally new approach based on primitive higher-dimensional structures and including new principles of reasoning not directly available in conventional foundations. Its applications include providing powerful and flexible computational tools that facilitate the large-scale formalization of mathematics.
Standard mathematical proofs are still just arguments in words; famous examples of erroneous published proofs abound. Recent advances in high-speed computing have permitted the development of powerful interactive theorem provers to aid in the verification of mathematical proofs. The current project employs a new interpretation of type theory recently discovered by the PI and Fields medalist Vladimir Voevodsky, which brings such systems much closer to everyday mathematical practice.
The PI also proposes to teach a graduate level course based on a current textbook in progress. He will also organize and lead a working group devoted to an ongoing formalization project developing a new interactive proof assistant for Homotopy Type Theory based on the popular Lean Proof assistant that is being developed by an international team based at Carnegie Mellon University.
These activities are intended to strengthen the interaction and cooperation between the Cambridge and CMU research groups involved in related work. Future exchanges and collaborations among faculty, postdoctoral, and doctoral researchers are expected to result.

Section 4 - Applicant Career Summary

Title of Current Position

Please state the title of your current position.

The Dean's Chair in Logic, Professor of Philosophy and Mathematics

Current Employer

Please enter the official organisation name of your current employer.

Carnegie Mellon University

Current Department

Departments of Philosophy and Mathematics

Contract Type

Please select your current contract type from the list below.

Permanent

Source of Salary

Please select your source of salary from the list below:

Overseas Source

Current Position Start Date

Please enter the date when your current position started.

01 August 1997

Current Position End Date

Please enter the date when your current position is expected to finish.

31 December 2050

Field of Specialisation

Please enter details of your field(s) of specialisation.

Category Theory, Mathematical Logic, Homotopy Type Theory

Personal Statement

Please provide a personal statement detailing your research career to date including prizes and achievements, and your career and research aspirations in the long term.

My research career divides roughly into two parts, separated by a discovery that I was fortunate to stumble across around 2005. Until then I had been focused on the areas in which I was trained in Chicago, namely, Category Theory under my advisor, the great Saunders Mac Lane, and Logic under the logician and philosopher W.W. Tait. My dissertation and early research involved mainly categorical and topological interpretations of various systems of logic, including for instance a topological completeness theorem for higher-order logic using sheaf theory. It was around this time that I completed my first textbook, Category Theory, which was published as an Oxford Logic Guide and is now widely used by students and non-specialists.

The pursuit of topological representations of ever more exotic logics led me finally to the challenge of Martin-Lof's constructive type theory during a visit to the Mittag-Leffler Institute in Sweden. In discussions there with Andrej Bauer, Ieke Moerdijk, and Erik Palmgren, a homotopical interpretation was conjectured as a natural extension of my previous work. This led to the idea of Homotopy Type Theory.

Shortly after working out the details of the homotopical interpretation with my student Michael Warren, I learned that the distinguished Fields medalist Vladimir Voevodsky at the Institute for Advanced Study (IAS) had independently arrived at a closely related idea at around the same time. I contacted Voevodsky and offered to exchange information and was delighted to find him eager to collaborate. There followed an intense period of interaction, with frequent reciprocal visits, IAS postdocs for my students, and joint research meetings, culminating in an IAS Special Year on Univalent Foundations, organized jointly by Voevodsky, Thierry Coquand, and myself. Leading researchers in Mathematics, Logic, and Computer Science were recruited from around the world to participate in a year-long effort to reshape the foundations of mathematics to accommodate computer verification. In addition to breakthrough mathematical results and libraries of formalized mathematics, the program produced a community of researchers, who together wrote the Homotopy Type Theory book in an unprecedented joint effort only made possible by tools developed for large-scale software development. The HoTT Book has been remarkably successful at disseminating these new ideas to the broader mathematical community, including specialists and non-specialists alike, from students to senior researchers. It has sold thousands of printed copies (at cost) and been downloaded many more thousands of times. In the twelve years since the IAS Special Year, numerous major research programs, international conferences, and smaller workshops, along with an online blog and email list (not to mention tens of millions of dollars in grant funding) have built a thriving ``HoTT research community''.

I am extremely fortunate to have found a research area that combines my interests and has been recognized by the wider scientific community (e.g. by making me Coordinating Editor of the Journal of Symbolic Logic). A Wolfson Fellowship will allow me to continue advancing this research program while passing on my knowledge to talented junior researchers at Cambridge and elsewhere in the UK.

Applicant Career History

Please list all of your appointments since your PhD and the dates in reverse chronological order, stating if part-time (and percentage part-time) when necessary.

Carnegie Mellon University:
Primary Appointment: Department of Philosophy, The Dean's Chair in Logic, 2024–present; Professor, 2008–2023; Assoc. Prof., 2002–2008; Asst. Prof., 1997–2002.
Secondary Appointment: Department of Mathematical Sciences, Professor, 2014–present.

Visiting Appointments:
- Hausdorff Institute for Mathematics, Bonn (Germany), Summer Semester 2024.
- Institute of Advanced Scientific Studies (IHES), Paris (France), June 2022.
- Center for Advanced Studies, Norwegian Academy of Science, Oslo (Norway), March–June 2019.
- Isaac Newton Institute for Mathematical Sciences, Cambridge University (UK), Summer Semester 2017.
- University of Stockholm (Sweden), Department of Mathematics, Spring Semester 2016.
- Institut Henri Poincaŕ e for Mathematical Research, Paris (France), Spring Semester 2014.
- Institute for Advanced Study (Princeton), Member, School of Mathematics, Academic Year 2012–13

CV

AwodeyWolfsonCV2025
07/03/2025
15:24:44
pdf 73.7 KB

Impact of Covid-19

The Society appreciates that the impact of the coronavirus pandemic on researchers and their work will be varied. Please provide a summary of how the pandemic has affected your research activities. (500 words max)

I used the time during the pandemic to undertake two quite large research projects that resulted in major works, which were subsequently submitted for publication but are not yet published (due to the delay involved in refereeing large technical works).
These therefore do not appear in my bibliography of published works. They are, however, available as preprints on the arXiv, namely:

- Cartesian cubical model categories, Steve Awodey, arXiv:2305.00893, 104pp., July 2023.

- The equivariant model structure on cartesian cubical sets, Steve Awodey, Evan Cavallo, Thierry Coquand, Emily Riehl, Christian Sattler, arXiv:2406.18497, 87pp., June 2024.

List your Key and/or Relevant Publications

1. Kripke-Joyal Forcing for Type Theory and Uniform Fibrations, (with N. Gambino, S. Hazratpour), Selecta Mathematica, 2024.
2. Impredicative Encodings of (Higher) Inductive Types, (with J. Frey and S. Speight), Logic in Computer Science (LICS), 2018.
3. Homotopy Initial Algebras in Type Theory, (with N. Gambino and K. Sojakova), Journal of the Association for Computing Machinery, 2017.
4. Natural Models of Homotopy Type Theory, Mathematical Structures in Computer Science, 2016.
5. Homotopy Type Theory: Unified Foundations of Mathematics and Computation, (with R. Harper), SIGLOG Newsletter, Association for Computing Machinery, 2014.
6. Voevodsky's Univalence Axiom in Homotopy Type Theory, (with Á. Pelayo and M. A. Warren), Notices of the American Mathematical Society, 2013.
7. Homotopy Type Theory and the Large-Scale Formalization of Mathematics, (with T. Coquand), The Institute

Letter, Institute for Advanced Study, 2013.
8. Inductive Types in Homotopy Type Theory, (with N. Gambino and K. Sojakova), Logic in Computer Science (LICS), 2011.
9. Homotopy-Theoretic Models of Identity Types, (with M. Warren), Mathematical Proceedings of the Cambridge Philosophical Society, 2009.
10. Topological Completeness for Higher-Order Logic, (with C. Butz), Journal of Symbolic Logic 65(3), 2000.

Applicant Research Funding

Please list all your current and previous research funding in reverse chronological order.

1. "Higher type theory", Air Force Office of Scientific Research, 2023–25, (700k. GBP).
2. "Homotopy Type Theory and the Formalization of Mathematics", Postdoctoral support, Army Research Office, 2021–23, (280k. GBP).
3. "Synthetic and Constructive Mathematics of Higher Structures in Homotopy Type Theory", Multi-Disciplinary University Research Initiative (MURI), Team Member, Air Force Office of Scientific Research, 2020–2026, (2m. GBP).
4. "Logical aspects of ∞-topoi", Postdoctoral support, Air Force Office of Scientific Research, 2020–2023, (350k. GBP).
5. "Homotopy Type Theory: Unified Foundations for Mathematics and Computation", Multi-Disciplinary University Research Initiative (MURI), Principal Investigator, Air Force Office of Scientific Research, 2014–2020, (6m. GBP).

Supporting documents

Please upload any documents (PDF), that you feel may support this application.

No Response

Section 5 - Research Proposal

Title

Please give the full title of your research proposal.

Homotopy Type Theory and the Formalization of Mathematics

Host Organisation

Please select the organisation where the award will be held from the drop down list below.

University of Cambridge

Host Department

Department of Computer Science and Technology

Start Date

Please enter the proposed start date

30 December 2025

End Date

Please enter the proposed end date for your research

29 December 2027

Scientific Categorisation

Physical Sciences

Subject Group

Computer Science

Subject Area RS

Please select one or more subject area(s) that most clearly defines the research area of the research proposal from the subject sub-category list below.

- ☑ Pure Mathematics
- ☑ Artificial Intelligence
- ☑ Programming languages and verification

Abstract

Please provide a scientific summary of your proposal. This should be a summary of the proposed research, briefly outlining the background and summarising the aims of your project

This proposal advances recent discoveries in the foundations of mathematics with far-reaching applications for the certainty and precision of mathematics, as well as for the everyday work of mathematicians and other scientists. Homotopy Type Theory is an emerging field combining logic, mathematics, and computer science and employing a fundamentally new approach to foundations based on primitive higher-dimensional structures and including new principles of reasoning not directly available in conventional foundations. Its applications range from allowing mathematicians to work invariantly with respect to a suitable notion of equivalence of structures, to directly expressing higher mathematical concepts such as infinity-categories and homotopy types, to providing powerful and flexible computational tools that facilitate the large-scale formalization of mathematics.

Standard mathematical proofs are still just arguments in words; famous examples of erroneous published proofs abound. In principle, proofs are assumed to be formally reducible to set theory, but in practice such reductions are so cumbersome as to be of little practical value. Recent advances in high- speed computing have permitted the development of powerful interactive theorem provers to aid in the verification of mathematical proofs. Rather than set theory, these systems use constructive type theory, the basic objects of which are structured data types, and the proofs in which permit the extraction of algorithmic information. Unfortunately, such type theories are unfamiliar to most mathematicians, and the systems based on them remain impractical for everyday use. The current project employs a homotopical interpretation of type theory recently discovered by the PI and Fields medalist Vladimir Voevodsky, which brings such systems much closer to everyday mathematical practice. This will permit the computer formalization of wide swaths of mathematics and has already led to impressive successes. It will also lead to the wide-spread use of computational proof assistants in pure and applied mathematics, creating powerful scientific tools.

The PI also proposes to teach a graduate level course on Type Theory, based on a current textbook in progress, and to organize and lead a working group devoted to the HoTT-Lean formalization project, a new interactive proof assistant for Homotopy Type Theory based on the Lean theorem prover, currently being developed by an team of researchers led by the PI at Carnegie Mellon. These activities are intended to strengthen the cooperation and interaction between the Cambridge and CMU research groups, resulting in future exchanges and collaborations among faculty, postdoctoral, and doctoral researchers.

Research Proposal

Please upload your proposal as a PDF file. PDF files must be no longer than 2 sides of A4, portrait orientation, be titled and the text size cannot be smaller than Arial size 10.

[WolfsonResearchProposalAwodey]
07/03/2025
04:44:58
pdf 124.6 KB

Outline of Data Management and Data Sharing Plan

If the proposed research will generate data that is of significant value to the research community, then please provide details of your data management and sharing plan.

The project will result in data of the following three kinds:

1. Theoretical results
2. Computer code
3. Documentation of the code

1. Theoretical results produced by the PI, either solely or in collaboration with UK researchers, will be made public according to standard practices in public lectures, in preprints posted on publicly accessible web pages, as preliminary versions posted on the public arXiv preprint server, and finally in articles submitted to scientific journals. Where possible, such published articles will be made available open access, and preliminary versions will remain publicly available on the arXiv.

2. Computer code will be produced in collaboration with other researchers in the UK and abroad according to standard practices, using publicly accessible, version controlled archiving tools (GitHub). Code will be distributed under an Open Source license, granting permission for others to use, modify, and share it for any purpose, subject to conditions preserving the provenance and openness of the software.

3. Documentation: All code produced as part of the project will be documented according to standard practices adopted by the relevant scientific community. Code for the Lean theorem prover will be contributed to the MathLib library, ensuring proper documentation as a condition of inclusion.

Fieldwork

Will you be conducting fieldwork as part of your research?

⦿ No

Section 6 - Use of Animals in Research

Does your proposal involve the use of animals or animal tissue?

⦿ No

Section 7 - Use of Human Participants, Patients or Tissue

Does your proposal involve the use of human participants, patients or tissue?

⦿ No

Section 8 - Financial Details

Please define the proposed budget for your project in the table below:

Budget heading		2025-2026			2026-2027
		Quarter 3: 2025 - 2026	Quarter 4: 2025 - 2026	Total	Quarter 1: 2026 - 2027
Bursary for Visiting Fellow					
Bursary for Visiting Fellow	Cost	£0.00	£0.00	£0.00	£0.00
Bursary for Visiting Fellow Total	***Cost***	***£0.00***	***£0.00***	***£0.00***	***£0.00***
Research Expenses					
Research Expenses	Cost	£21,500.00	£7,000.00	£28,500.00	£8,500.00
Research Expenses Total	***Cost***	***£21,500.00***	***£7,000.00***	***£28,500.00***	***£8,500.00***
Relocation and Visa Costs					
Relocation and Visa Costs	Cost	£1,751.00	£0.00	£1,751.00	£0.00
Relocation and Visa Costs Total	***Cost***	***£1,751.00***	***£0.00***	***£1,751.00***	***£0.00***
Grand Total	**Cost**	**£23,251.00**	**£7,000.00**	**£30,251.00**	**£8,500.00**

Budget heading		2026-2027			
		Quarter 2: 2026 - 2027	Quarter 3: 2026 - 2027	Quarter 4: 2026 - 2027	Total
Bursary for Visiting Fellow					
Bursary for Visiting Fellow	Cost	£0.00	£0.00	£0.00	£0.00
Bursary for Visiting Fellow Total	***Cost***	***£0.00***	***£0.00***	***£0.00***	***£0.00***
Research Expenses					
Research Expenses	Cost	£18,000.00	£0.00	£0.00	£26,500.00
Research Expenses Total	***Cost***	***£18,000.00***	***£0.00***	***£0.00***	***£26,500.00***
Relocation and Visa Costs					
Relocation and Visa Costs	Cost	£0.00	£1,035.00	£0.00	£1,035.00
Relocation and Visa Costs Total	***Cost***	***£0.00***	***£1,035.00***	***£0.00***	***£1,035.00***

Budget heading		2026-2027			
		Quarter 2: 2026 - 2027	Quarter 3: 2026 - 2027	Quarter 4: 2026 - 2027	Total
Grand Total	**Cost**	**£18,000.00**	**£1,035.00**	**£0.00**	**£27,535.00**

Budget heading		2027-2028			Total
		Quarter 1: 2027 - 2028	Quarter 2: 2027 - 2028	Total	
Bursary for Visiting Fellow					
Bursary for Visiting Fellow	Cost	£0.00	£0.00	£0.00	£0.00
Bursary for Visiting Fellow Total	***Cost***	***£0.00***	***£0.00***	***£0.00***	**£0.00**
Research Expenses					
Research Expenses	Cost	£10,000.00	£19,000.00	£29,000.00	£84,000.00
Research Expenses Total	***Cost***	***£10,000.00***	***£19,000.00***	***£29,000.00***	**£84,000.00**
Relocation and Visa Costs					
Relocation and Visa Costs	Cost	£0.00	£0.00	£0.00	£2,786.00
Relocation and Visa Costs Total	***Cost***	***£0.00***	***£0.00***	***£0.00***	**£2,786.00**
Grand Total	**Cost**	**£10,000.00**	**£19,000.00**	**£29,000.00**	**£86,786.00**

Justification for Research Expenses

-Budget includes cost of living and mobility expenses of £78,000 which are to cover costs for a serviced apartment in Cambridge (allowing for costs of up to £5,000 per month x12 months), costs of subsistence and other living costs required to reside in the Cambridge area for the fellowship (allowing for costs of up to £1,000 per month), and also £6,000 to cover 3 round-trip airfares for the PI and one for the spouse over the duration of the grant (24 months), transportation to and from airports, and any additional costs associated with such travel.

-Travel within the UK assumes 5, week-long, research trips/year to other sites in the UK for the purpose of collaboration and dissemination, including rail or air travel and local accommodation (no meals).

-An additional cost of £1,000 has been budgeted to allow for shipping of books to aid in research activities.

Justification for Relocation and Visa expenses

Please include the number of dependents (partner and children only allowed), which visa type you are applying for, its cost, and an estimation of other relocation expenses.

-Budget includes £2,786 to cover cost of Global Talent Visa (£716) and 2x annual NHS Surcharge (2x £1,035).

Section 9 - Applicant Declaration

Declaration

I hereby declare that the information provided in this application is true and correct to the best of my knowledge.

Checked

Applicant Name	Steve Awodey
Date	06 March 2025

CURRICULUM VITAE

Steve Awodey, The Dean's Chair in Logic, Carnegie Mellon University

Academic Position

Carnegie Mellon University

Department of Philosophy (Primary): *The Dean's Chair in Logic*, 2024–present
Professor, 2008–2023; *Assoc. Prof.*, 2002–2008; *Asst. Prof.*, 1997–2002.

Department of Mathematical Sciences (Secondary): *Professor*, 2014–present.

Visiting Appointments

1. Hausdorff Institute for Mathematics, Bonn, Summer Semester 2024.
2. Institute of Advanced Scientific Studies (IHÉS), Paris, June, 2022.
3. Center for Advanced Studies, Norwegian Academy of Science, Oslo, Spring Semester 2019.
4. Isaac Newton Institute, Cambridge University, Summer Semester 2017.
5. University of Stockholm, Department of Mathematics, Spring Semester 2016.
6. Institut Henri Poincaré for Mathematical Research, Paris, Spring Semester 2014.
7. Institute for Advanced Study, Princeton, School of Mathematics, Academic Year 2012–13

Educational History

The University of Chicago: *Ph.D. Philosophy* (with honors), 1997.
Supervised by S. Mac Lane (Mathematics) and W.W. Tait (Philosophy). *M.Sc. Mathematics*, 1992.

Philipps-Universität Marburg, Germany: *Magister Artium*, 1989.

Selected Publications

Books

1. *Homotopy Type Theory: Univalent Foundations of Mathematics*, The Univalent Foundations Program, Institute for Advanced Study, 2013.
2. *Category Theory*, S. Awodey, Oxford Logic Guides 49, Oxford University Press, 2006. 2nd edition, Oxford Logic Guides 52, 2010.

Articles

1. Kripke-Joyal Forcing for Type Theory and Uniform Fibrations, (with N. Gambino, S. Hazratpour), *Selecta Mathematica*, 2024.
2. Impredicative Encodings of (Higher) Inductive Types, (with J. Frey and S. Speight), *Logic in Computer Science (LICS)*, 2018.
3. Homotopy Initial Algebras in Type Theory, (with N. Gambino and K. Sojakova), *Journal of the Association for Computing Machinery*, 2017.
4. Natural Models of Homotopy Type Theory, *Mathematical Structures in Computer Science*, 2016.
5. Homotopy Type Theory: Unified Foundations of Mathematics and Computation, (with R. Harper), *SIGLOG Newsletter*, Association for Computing Machinery, 2014.

6. Voevodsky's Univalence Axiom in Homotopy Type Theory, (with Á. Pelayo and M. A. Warren), *Notices of the American Mathematical Society*, 2013.
7. Homotopy Type Theory and the Large-Scale Formalization of Mathematics, (with T. Coquand), *The Institute Letter*, Institute for Advanced Study, 2013.
8. Inductive Types in Homotopy Type Theory, (with N. Gambino and K. Sojakova), *Logic in Computer Science (LICS)*, 2011.
9. Homotopy-Theoretic Models of Identity Types, (with M. Warren), *Mathematical Proceedings of the Cambridge Philosophical Society*, 2009.
10. Topological Completeness for Higher-Order Logic, (with C. Butz), *Journal of Symbolic Logic*, 2000.

Selected Invited Lectures

1. "Homotopical Semantics of Type Theory" (3 lectures), Workshop on Interactions of Proof Assistants and Mathematics, University of Regensburg, Germany, September 2023.
2. "Intensionality, Invariance, and Univalence", 2019 Skolem Lecture, Department of Philosophy, University of Oslo, May 2019.
3. "An Overview of Homotopy Type Theory", Felix Hausdorff Commemorative Colloquium, Hausdorff Center for Mathematics, Bonn, Germany, June 2018.
4. "Impredicative Encodings in Homotopy Type Theory", Invited Plenary Lecture, *Big Proof*, Newton Institute for Mathematics, Cambridge University, UK, July 2017.
5. "A Cubical Model of Homotopy Type Theory", Invited series of four lectures in Logic and Topology, Department of Mathematics, University of Stockholm, May–June 2016.
6. "Homotopy Type Theory", Hari Sahasrabuddhe Inflections in Computer Science Lecture, Indian Institute of Technology, Kanpur, India, January 2015.
7. "Univalence as a New Principle of Logic", Inaugural Lecture, Calgary Mathematics and Philosophy Lecture Series, October 2014.
8. "Advances in Homotopy Type Theory", Invited Plenary Lecture, TYPES 2014, Institut Henri Poincare, Paris, May 2014.
9. "Homotopy Type Theory and Univalent Foundations", Invited Plenary Lecture, Quantum Logic and Computation, Clay Mathematics Institute, Oxford University, September 2013.
10. "Advances in Homotopy Type Theory", Invited Plenary Lecture, LICS and MFPS, New Orleans, June 2013.
11. "Homotopy Type Theory and Univalent Foundations", Invited Plenary Lecture, TYPES 2013, Tolouse, France, April 2013.
12. "Natural Models of Type Theory", Univalent Foundations Seminar, Institute for Advanced Study, April 2013.
13. "Univalent Foundations", Members Seminar, School of Mathematics, Institute for Advanced Study, December 2012.
14. "Constructive Type Theory and Homotopy Theory", School of Mathematics, Institute for Advanced Study, December 2010.

Recent Major Awards and Grants

1. Air Force Office of Scientific Research, "Higher Type Theory", (joint with Robert Harper), 2023–2025, ($900,000).
2. Army Research Office, "Homotopy Type Theory and the Formalization of Mathematics," Postdoctoral Support, 2021–23, ($360,000).
3. Air Force Office of Scientific Research, "Synthetic and Constructive Mathematics of Higher Structures in Homotopy Type Theory," Multi-Disciplinary University Research Initiative (MURI), Team Member, 2020–2026, ($2.5 mil.).
4. Air Force Office of Scientific Research, "Logical Aspects of ∞-Topoi," Postdoctoral Support, 2020–2023, ($450,000).
5. Air Force Office of Scientific Research, "Homotopy Type Theory: Unified Foundations for Mathematics and Computation," Multi-Disciplinary University Research Initiative (MURI), Principal Investigator, 2014–2020, ($7.5 mil.).

Editorial Activities

Journal of Symbolic Logic: Coordinating Editor, 2023–present; Editor, 2021–23.

Editorial Board Member: *Mathematical Logic Quarterly*, *Philosophia Mathematica*, *Bulletin of Symbolic Logic* (Reviews Managing Editor 2008–2013), *Review of Symbolic Logic* (Advisory Board).

Professional Activities

Scientific committee member and Chair, *International Conference on Category Theory (CT 2019)*, University of Edinburgh, Scotland, July 2019.

Program committee member and local organizing committee Chair, *International Conference on Homotopy Type Theory (HoTT 2019)*, Carnegie Mellon University, Pittsburgh, August 2019.

Co-Organizer (with Thierry Coquand and Vladimir Voevodsky) of a Special Year on *Univalent Foundations of Mathematics*, Institute for Advanced Study, Princeton, 2012–13.

Scientific Mentorship

Postdoctoral Advisor: Reid Barton 2023-25; Mathieu Anel 2018–25; Jonas Frey 2017–24; Andrew Swan 2019–22; Felix Wellen 2018-20; Ulrik Buchholtz 2016–17; Bas Spitters 2014–15.

Doctoral Advisor: 12 Ph.D. students in Pure and Applied Logic, Mathematics, and Computer Science at Carnegie Mellon University and elsewhere.

Royal Society Wolfson Visiting Fellowship Research Proposal

HOMOTOPY TYPE THEORY AND THE FORMALIZATION OF MATHEMATICS

Steve Awodey, The Dean's Chair in Logic, Carnegie Mellon University

Summary of the Proposed Research

This proposal advances a recent breakthrough in the foundations of mathematics with far-reaching practical applications for the certainty and precision of mathematical results and even for the day-to-day work of mathematicians and other scientists. *Homotopy Type Theory* (HoTT) is an emerging field at the intersection of logic, mathematics, and computer science that is reshaping the foundations of those disciplines [24]. It is a fundamentally new approach based on primitive higher-dimensional structures, and admitting new principles of reasoning not directly available in the conventional foundation based on set theory. Its applications range from allowing mathematicians to work invariantly with respect to equivalence of structures, to directly expressing higher mathematical concepts such as ∞-categories and homotopy types, to offering powerful and flexible tools that will facilitate the computerized formalization of mathematics.

Successful completion of the research proposed here will contribute to the future wide-spread use of computational proof assistants in pure and applied mathematics and promote in the large-scale formalization of mathematics, enabling the creation of powerful scientific tools.

Identification of the PI

The conceptual basis of this discovery is a new homotopical semantics for the constructive type theories on which modern computer proof assistants are based, which was discovered independently by the PI and the late Fields medalist V. Voevodsky [9, 27]. Its feasibility was tested and demonstrated during a special year at the Institute for Advanced Study (Princeton) in 2012–13, which was organized by the PI, Voevodsky, and leading computer scientist T. Coquand (Gothenburg, Sweden) [5]. Since that seminal event, the PI has been one of the leaders of an emerging international scientific community consisting of researchers in computer science, logic, algebraic topology, and category theory. He led a major DoD-funded research project in 2014–19 ($9M), was a team member in a similar-sized follow-up project in 2020–2025, and has been involved in several major research programs at leading international research institutes. Two international conferences for HoTT were co-organized by the PI and held at CMU in 2019 and 2023, with each gathering over 100 international participants.

Technical Description

Today, even the most rigorous and precise mathematical proofs are still just arguments in *words*. Famous examples of erroneous published proofs abound, including widely accepted results by leading experts, published in distinguished refereed journals [28]. Conventionally, mathematical proofs are assumed to be formally reducible to Zermelo-Frankel *set theory*, thus providing a higher degree of certainty; but in practice such reductions are so lengthy, and so difficult to actually perform by hand, that this foundation is of no practical use for establishing the correctness of mathematical results. In the last two decades, advances in high-speed computing have permitted the development of increasingly powerful computerized proof systems that can aid in the formalization and verification of mathematical proofs. In place of set theory these systems often use *constructive type theory*, the basic objects of which are structured data types consisting of lists, trees, functions, etc., and which allow the extraction of algorithmic information from the proof terms constructed [20]. This is an advance, but the methods and results attainable by such systems are still too far from everyday mathematical practice to be of actual use: such types and terms are unfamiliar to working mathematicians. Moreover, mathematicians routinely identify isomorphic objects, but in the current generation of proof assistants such identifications must still be laboriously encoded, presenting a major obstacle to the widespread adoption of such tools. The recent lengthy proof of the Feit-Thompson odd-order theorem by a team of researchers using the Coq proof assistant is one case in point [15]. Even the more recent, and much more sophisticated, Liquid Tensor Experiment in the Lean proof system [21] still does not employ invariant methods that permit automatic transport of structure across equivalences.

In HoTT, by contrast, types are interpreted not just as structured set-like objects, but as abstract *spaces* as in homotopy theory [1]. In particular, the fundamental identity type $x =_A y$, which traditionally represents the type of proofs that two terms x and y of the type A are equal, is identified with the space of all continuous paths between points x and y in a space A; and unlike the mere fact that x and y are equal, this path-space may have an intricate structure encoding essential information about the type A. This is just one instance of a fruitful interaction between type theory and homotopy theory, which also manifests itself in the modeling of dependent types as fibrations, inductive types as homotopy colimits, and universes as classifying spaces. Under this new point of view, homotopy theory suggests fundamental improvements to type theory, solving long-standing foundational problems. With these improvements, type theory can directly describe homotopical and higher-categorical objects, which have growing applications throughout mathematics, opening up new areas of mathematics to computer-assisted proofs. Finally, these improvements also simplify the formalization of standard (non-homotopical) mathematics by permitting invariant reasoning in general, vastly expanding the range of feasible formalization.

One such improvement is Voevodsky's profound *univalence axiom* [8], which states that for a universe type $\mathcal{U}$, whose elements X, Y are types, the identity type $X =_{\mathcal{U}} Y$ is equivalent to the type $X \simeq Y$ of equivalences between the types X and Y; briefly,

$$(X =_{\mathcal{U}} Y) \simeq (X \simeq Y). \tag{UA}$$

This imports the idea of classifying spaces from homotopy theory and formally justifies the identification of isomorphic structures, for it yields a function $(X \simeq Y) \to (X =_{\mathcal{U}} Y)$ that transforms an isomorphism into an *equality*. Univalence is unavailable in set theoretic foundations, but it has been shown to be formally compatible with the type theory used in modern proof assistants such as Coq, Agda, and Lean [10].

Research Directions

The proposed research falls into the three main areas of *semantics*, *syntax*, and *computation*, in each of which a new direction is pursued, namely: the effective ∞-topos, algebraic type theory, and the development of a new HoTT version of the Lean theorem prover. The selection of these particular avenues of investigation is determined by recent advances due to the PI and his collaborators and judged to have the potential for major breakthroughs, as briefly indicated below.

$*$ *Semantics: The effective ∞-topos.* Type theoretic ideas have recently been employed by the PI and collaborators to construct new Quillen model categories presenting ∞-toposes [19] with specified properties [3, 4]. Using this approach, one can also construct an (elementary) ∞-topos based on recursive realizability, which internalizes computation in the same way as does Hyland's celebrated "effective topos" [14]. This semantic construction includes a universe of types which is at once *univalent* and *complete*, or "impredicative", a combination which is not even consistent with classical mathematics, but provides a powerful extension of conventional HoTT in which one can construct sophisticated and complex, recursively defined structures [6].

$*$ *Syntax: Algebraic type theory.* Such homotopical and (higher) categorical models do not admit a direct interpretation of the formal syntax of dependent type theory; rather, they must first be "strictified" in one of several known ways [2], in order to faithfully model the rigid formal rules of type theory. One approach to such strictification pioneered by the PI proceeds via an axiomatization of the syntax of dependent type theory in categorical algebra, using the formalism of polynomial functors [7]. This approach is especially well-suited to homotopical interpretations, because it is essentially "equation-free" and so does not give rise to the coherence problems inherent in conventional algebraic approaches involving (an infinite hierarchy of) equations. A related approach was used in M. Shulman's milestone proof that every Grothendieck ∞-topos admits a univalent universe [22], and it plays an important practical role in the HoTT-Lean formal system introduced next.

$*$ *Computation: HoTT-Lean.* Past computer proof systems developed for Homotopy Type Theory include Coq-HoTT [11], Agda-UF [23], Cubical Agda [26], and others [25]. A HoTT mode was implemented in early versions of Lean (2 and 3), mainly by a team working at CMU [13]. The current version of the Lean proof assistant (Lean 4, [17]) is rapidly becoming the system of choice for working mathematicians eager to employ the new tools of formalization currently becoming available (see for instance [18]). This proposal advances a new HoTT-Lean proof system on the basis of cubical and higher groupoid semantics and algebraic type theory. It not only supports powerful synthetic reasoning but also provides an automatic translation of the results into classical foundations, along with a verification of their correctness – a first for implementations of HoTT.

Collaboration and Diffusion

The PI will undertake the following collaborative activities and outreach as part of the research effort.

1. Enhance and strengthen existing research ties with host M. Fiore (Computer Science, Cambridge), specifically in the area of algebraic type theory and abstract syntax.

2. Develop new joint research directions with Fiore and past collaborator N. Gambino (Mathematics, University of Manchester) in the areas of (higher) topos theory, type theory, and categorical logic.

3. Disseminate the latest research advances by lecturing from a textbook currently in progress on Categorical Logic and Type Theory and by leading a working group on the HoTT-Lean project.

References

[1] S. Awodey. Type theory and homotopy. In *Essays on the Foundations of Mathematics in Honor of Per Martin-Löf*, P. Dybjer et al. (ed.s). Springer, 2012.

[2] S. Awodey. Natural models of homotopy type theory. *Mathematical Structures in Computer Science*, 2016.

[3] S. Awodey. Cartesian cubical model categories. arXiv:2305.00893, 2024.

[4] S. Awodey, E. Cavallo, T. Coquand, E. Riehl, and C. Sattler. The equivariant model structure on cubical sets. arXiv:2406.18497, 2024.

[5] S. Awodey and T. Coquand. Univalent foundations and the large-scale formalization of mathematics. Institute for Advanced Study, *The Institute Letter*, Summer 2013.

[6] S. Awodey, J. Frey and S. Spieght. Impredicative encodings of (higher) inductive types. LICS 2018.

[7] S. Awodey and C. Newstead. Polynomial pseudomonads and dependent type theory. arXiv:1802.00997, 2018.

[8] S. Awodey, A. Pelayo and M. Warren. Voevodsky's univalence axiom in Homotopy Type Theory. *Notices of the AMS*, 60(9):1164–67, 2013.

[9] S. Awodey and M.A. Warren. Homotopy theoretic models of identity types. Mathem. Proc. of the Cambridge Philos. Soc., 146:45–55, 2009.

[10] C. Cohen, T. Coquand, S. Huber, and A. Mörtberg. *Cubical type theory: A constructive interpretation of the univalence axiom*, TYPES 2015, 5:1–5:34, 2015.

[11] Coq-HoTT: A Coq library for Homotopy Type Theory. `https://github.com/HoTT/Coq-HoTT`, 2025.

[12] F. van Doorn. Spectral sequences in homotopy type theory. Carnegie Mellon University. June 4, 2018.

[13] F. van Doorn and L. De Moura. The Lean Homotopy Type Theory Library. `https://github.com/leanprover/lean2/blob/master/hott/hott.md`, 2016.

[14] J.M.E. Hyland. The effective topos. The L.E.J. Brouwer Centenary Symposium Edited by A.S. Troelstra, D. van Dalen Studies in Logic and the Foundations of Mathematics, vol. 110, pp. 165–216, 1982.

[15] Gonthier, G., et al. A machine-checked proof of the odd order theorem. *ITP 2013*, *LNCS* vol. 7998, 2013.

[16] A. Joyal. The theory of quasi-categories and its applications. Centre de Recerca Matemàtica, 2008.

[17] Lean: Programming Language and Theorem Prover. `https://lean-lang.org`, 2025.

[18] Lean Community: Lean and its Mathematical Library. `https://leanprover-community.github.io`, 2025.

[19] J. Lurie. *Higher topos theory*. Princeton University Press, 2009.

[20] P. Martin-Löf. Constructive mathematics and computer programming. Department of Mathematics, University of Stockholm, 1979.

[21] The Mathlib community. *Completion of the Liquid Tensor Experiment*. `https://leanprover-community.github.io/blog/posts/lte-final`, 2022.

[22] M. Shulman. All $(\infty, 1)$-toposes have strict univalent universes. arXiv.1904.07004, 2019

[23] The Agda Unimath Library. `https://unimath.github.io/agda-unimath`, 2025.

[24] The Univalent Foundations Program. *Homotopy type theory: Univalent foundations for mathematics*. Institute for Advanced Study, `www.homotopytypetheory.org/book`, 2013.

[25] UniMath: Univalent Mathematics in the Rocq proof assistant. `https://unimath.github.io/UniMath`, 2025.

[26] A. Vezzosi, A. Mörtberg, and A. Abel. Cubical Agda: A dependently typed programming language with univalence and higher inductive types. Proceedings of the ACM on Programming Languages, 3(87), pp. 1–29, 2019.

[27] V. Voevodsky. A very short note on homotopy λ-calculus. Unpublished, 2006.

[28] V. Voevodsky. The origins and motivations of univalent foundations. Institute for Advanced Study, *The Institute Letter*, 2014.